Shift Radix Systems and Variations of Them

Paul Surer[1] <paul.surer@reflex.at>

For $\mathbf{r} \in \mathbb{R}^{\mathbf{d}}$ define the mapping $\tau_{\mathbf{r}} : \mathbb{Z}^d \to \mathbb{Z}^d, \mathbf{x} = (x_1, \ldots, x_d) \mapsto (x_2, \ldots, x_d, -\lfloor \mathbf{r} \cdot \mathbf{x} \rfloor)$. $\tau_{\mathbf{r}}$ is called a shift radix system (SRS) if $\forall \mathbf{x} \in \mathbb{Z}^d : \exists n \in \mathbb{N}$ with $\tau_{\mathbf{r}}^n(\mathbf{x}) = \mathbf{0}$. Shift radix systems are strongly related to well known notions of number systems. Let $\mathcal{D}_d^0 := \{r \in \mathbb{R}^d | \tau_{\mathbf{r}} \text{ is SRS}\}$. We will present results concerning the characterisation of $\mathcal{D}_2^0$. Furthermore we will consider the following variant of SRS: Let $\tilde{\tau}_{\mathbf{r}}(\mathbf{x}) = (x_2, \ldots, x_d, -\lfloor \mathbf{r} \cdot \mathbf{x} + \frac{1}{2} \rfloor)$. The mapping $\tilde{\tau}_{\mathbf{r}}$ is called a symmetric shift radix system (SSRS), if $\forall \mathbf{x} \in \mathbb{Z}^d : \exists n \in \mathbb{N}$ with $\tilde{\tau}_{\mathbf{r}}^n(\mathbf{x}) = \mathbf{0}$. The set of SSRS parameters is defined by $\tilde{\mathcal{D}}_d^0 := \{r \in \mathbb{R}^d | \tau_{\mathbf{r}} \text{ is SSRS}\}$. This set can be completely characterised for $d = 2$. For $d = 3$ we will discuss some new characterization results. Supported by FWF Project Nr. P17557-N12.

[1]Montanuniversität Leoben