# Computational Micromagnetism for Large-Soft Magnets

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The large body limit in the Landau-Lifshitz equations of micromagnetics yields a macroscopic model without exchange energy and convexified side conditions for the macroscopic magnetisation vectors. Its Euler Lagrange equations ***(P)*** read: Given a magnetic body $\Omega \subseteq \mathbb{R}^d$, $d = 2, 3$, an exterior field $\mathbf{f} \in L^2(\Omega)^d$ and the convexified anisotropy density $\phi^{**} : \mathbb{R}^d \to \mathbb{R}_{\geq 0}$, find a magnetization $bfm \in L^2(\Omega)^d$ and a Lagrange multiplier $\lambda \in L^2(\Omega)$ such that a.e. in $\Omega$

$$\begin{aligned}
&\nabla u + D\phi^{**}(\mathbf{m}) + \lambda \mathbf{m} = \mathbf{f},\\
&|\mathbf{m}| \leq 1, \quad \lambda \geq 0,\\
&\lambda(1 - |\mathbf{m}|) = 0.
\end{aligned}$$

The potential $u \in H^1_{\ell oc}(\mathbb{R}^d)$ solves the (Maxwell) equations

$$\nabla u \in L^2(\mathbb{R}^d)^d \text{ and } \operatorname{div}(-\nabla u + \mathbf{m}) = 0 \text{ in } \mathcal{D}'(\mathbb{R}^d)$$

in the entire space. It therefore appears natural to recast the associated far field energy into an integral operator $\mathcal{P}$ which maps $\mathbf{m}$ to the corresponding stray field $\nabla u = \mathcal{P}\mathbf{m}$.
The proposed numerical scheme involves the operator $\mathcal{P}$ and replaces pointwise side-condition $|\mathbf{m}| \leq 1$ by a penalization strategy. Given a triangulation $\mathcal{T}$, the induced space of piecewise constant functions $P_0(\mathcal{T})$ on $\Omega$, and a penalization parameter $\varepsilon > 0$, the discrete penalized problem $(\boldsymbol{P_{\varepsilon,h}})$ reads: Find $\mathbf{m}_h \in P_0(\mathcal{T})^d$ such that for all $\boldsymbol{\nu}_h \in P_0(\mathcal{T})^d$

$$\langle \mathcal{P}\mathbf{m}_h + D\phi^{**}(\mathbf{m}_h) + \lambda_h \mathbf{m}_h \,; \boldsymbol{\nu}_h \rangle_{L^2(\Omega)} = \langle \mathbf{f} \,; \boldsymbol{\nu}_h \rangle_{L^2(\Omega)}$$

with $\lambda_h := \varepsilon^{-1} \dfrac{\max\{0, |\mathbf{m}_h| - 1\}}{|\mathbf{m}_h|} \in P_0(\mathcal{T})$.
Numerical aspects addressed in the presentation include the efficient treatment of $\mathcal{P}$ via hierarchical matrices as well as a priori and a posteriori error control and adaptive mesh-design.
*The talk is based on joint work with Carsten Carstensen (Humboldt-Universität Berlin) and Nikola Popović (University of Boston).*

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