

# Nested Reversal Schedules

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The iterative solution of optimal control problems in ODEs by various methods leads to a succession of triple sweeps through the discretized time interval. The second (adjoint) sweep relies on information from the first (original) sweep, and the third (final) sweep depends on both of them. Typically the steps on the adjoint sweep involve more operations and require more storage than the other two. In order to avoid storing full traces of the original and adjoint sweeps we consider nested reversal schedules that require only the storage of selected original and adjoint intermediate states called thin and fat checkpoints. The schedules are designed to minimize the overall execution time given a certain total amount of storage for the checkpoints.

We consider the optimal solution for this discrete optimization problem and develop a cheap heuristic for constructing nested reversals that are quite close to optimality. Moreover, we demonstrate that the dependence on  $l$  can be arranged polylogarithmically by nested checkpoint strategies. Consequently, the operations count also grows as a second power of  $\log_C l$ , which needs not result in an increase of the actual run-time due to memory effects.

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