Substructures of Hom, especially some related to regularity

Friedrich Kasch[1] <friedrich.kasch@t-online.de>

We consider the category of unitary right R-modules for a ring R with $1 \in R$. Let

$$H := \mathrm{Hom}_R(A, M), \quad S := \mathrm{End}(M_R), \quad T := \mathrm{End}(A_R).$$

Then H is an S-T-bimodule.

Some years ago I started to study the substructures of Hom, mainly the connections between them. In the first years I had a very stimulating cooperation with K.I. Beidar, Taiwan. All this material and more was published in the book "Rings, Modules, and the Total" by A. Mader, Honolulu, and myself, Birkhäuser Verlag, 2004. A. Maderand I continued our cooperation with the program to study the different regular substructures of Hom. Fundamentals for this topic are contained in the following two papers: "Regularity in Hom" by F. Kasch, Verlag Reinhard Fischer, 1996, and "Regularity and substructures of Hom"by A. Mader and myself, to appear in Communications in Algebra. The interesting topic for regularity in Hom is the fact that in Hom there exist several definitions of regularity and for each there exist maximal regular submodules of $\mathrm{Hom}_R(A, M)$. For these maximal regular submodules we have for example that

$$\mathrm{Reg}(A, M) \subseteq \text{Semi-Reg}(A, M) \subseteq \mathrm{Rad}(H)\text{–}\mathrm{Reg}(A, M).$$

Details will be given in my talk. Here I mention only that $\mathrm{Reg}(A, M)$ is the unique largest regular S-T-submodule for ordinary regularity, semi-regularity is a generalization of semi-regularity for modules (studied by Nicholson), and Rad-regularity is relative regularity defined and considered in my paper mentioned above.

Obviously, there are many questions about the "Regs" dependent on A and M. For which A and M are the Regs equal to 0 or to H? Which Regs are contained in or equal to others? What are the relations of the Regs with other substructures of Hom? For example, is $\mathrm{Reg}(A, M) \cap \mathrm{Tot}(A, M) = 0$ for all A and M? This leads to the question: For which A and M is $H = \mathrm{Reg}(A, M) \oplus \mathrm{Tot}(A, M)$?

[1]LMU München