Optimizing the Algebraic Connectivity of a Graph

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Fiedler proved that the second smallest eigenvalue of the Laplace matrix of a graph is tightly related to the connectivity of the graph and coined the name algebraic connectivity. Eigenvectors to this eigenvalue are frequently used in graph bisection heuristics, yet mathematical insight into these interrelations is still weak. In order to gain a better understanding, we study the problem of maximizing the algebraic connectivity over all weighted graphs with the same edge set and bounded total weight. Using semidefinite programming techniques and exploiting optimality conditions we show that the latter problem is equivalent to finding an embedding of the n nodes in $n-$space so that their barycenter is at the origin, the distance between adjacent nodes is bounded by one and the nodes are spread as much as possible (the sum of the squared norms is maximized). Based on this interpretation, the optimal embedding and the optimized algebraic connectivity can be computed directly for trees. For general graphs we prove that in optimal embeddings the barycenters of partitions induced by separators are separated by the affine subspace spanned by the nodes of the separator.

[1] M. Fiedler: *Laplacian of graphs and algebraic connectivity*, Combinatorics and Graph Theory 25:57–70, 1989

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