Reduced-Basis Approximation and A Posteriori Error Estimation for Parabolic Partial Differential Equations

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Modern engineering problems often require accurate, reliable, and efficient evaluation of quantities of interest, evaluation of which demands the solution of a partial differential equation. In this talk we present a technique for the predicition of outputs of interest of parabolic partial differential equations.

The essential ingredients are: (*i*) rapidly convergent reduced-basis approximations – Galerkin pro jection onto a space W_N spanned by solutions of the governing partial differential equation at N selected points in parameter-time space; (*ii*) *a posteriori* error estimation – relaxations of the error-residual equation that provide rigorous and sharp bounds for the error in specific outputs of interest; and (*iii*) offline-online computional procedures. The operation count for the online stage depends only on N (typically small) and the parametric complexity of the problem; the method is thus ideally suited for repeated, rapid, reliable evaluation of input-output relationships in the many-query or real-time contexts.

We present numerical examples to illustrate the applicability of our methods in the many-query contexts of optimization, characterization, and control. We consider several applications: the non-destructive evaluation of delamination in fiber-reinforced concrete, the dispersion of pollutants in a rectangular domain, and the control of welding quality.

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